Question 4 (8 marks)

The planes with equations and intersect at point .

(a) Determine the coordinates of point . (3 marks)

Points and have coordinates and respectively.

(b) Determine the vector equation of the straight line through points and in the  
form . (2 marks)

(c) Determine the Cartesian equation of the plane that contains point , point and the origin. (3 marks)

Question 4 (8 marks)

The planes with equations and intersect at point .

(a) Determine the coordinates of point . (3 marks)

|  |
| --- |
| Solution |
| Hence . |
| Specific behaviours |
| ✓ solves for one variable  ü solves for second variable  ü states correct coordinates |

Points and have coordinates and respectively.

(b) Determine the vector equation of the straight line through points and in the  
form . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ obtains  ü correct equation in required form |

(c) Determine the Cartesian equation of the plane that contains point , point and the origin. (3 marks)

|  |
| --- |
| Solution |
| Hence equation of plane is . |
| Specific behaviours |
| ✓ forms cross product using two vectors in plane  ü obtains normal  ü states correct equation |

Question 1 (8 marks)

The equations of planes and are shown below.

(a) Show that the equation of plane in Cartesian form is . (3 marks)

(b) The origin lies on the surface of sphere . Determine the vector equation of , given that its centre is the point of intersection of the three planes. (5 marks)

Question 1 (8 marks)

The equations of planes and are shown below.

(a) Show that the equation of plane in Cartesian form is . (3 marks)

|  |
| --- |
| Solution |
| Cross product of vectors in plane is .  Let normal to plane be , so that .  Hence Cartesian equation is . |
| Specific behaviours |
| ✓ writes cross product of two vectors that lie in  ü obtains normal to plane  ü uses vector equation to derive Cartesian equation |

(b) The origin lies on the surface of sphere . Determine the vector equation of , given that its centre is the point of intersection of the three planes. (5 marks)

|  |
| --- |
| Solution |
| Subtracting last from first gives .  Subtracting last from middle gives .  Hence and so intersect at .  Distance from centre of to is .  Hence equation of is . |
| Specific behaviours |
| ✓ writes system of equations, correctly starts elimination  ü solves for one variable  ü solves for all variables  ü determines distance from origin  ü correctly writes vector equation of sphere |

Question 3 (6 marks)

Consider the system of equations (where is a constant) given by

(a) When , solve the system of equations and interpret your solution geometrically.

(4 marks)

(b) State the value of for which the system of equations has no solution and explain the geometric interpretation of this. (2 marks)

Question 3 (6 marks)

Consider the system of equations (where is a constant) given by

(a) When , solve the system of equations and interpret your solution geometrically.

(4 marks)

|  |
| --- |
| Solution |
| Eliminate using :  Substitute for and then :  Substitute for and then:  Hence the system is three planes that intersect at the point . |
| Specific behaviours |
| ✓ eliminates a variable correctly  ü solves for one variable  ü solves for all variables  ü states three planes intersecting at a point |

(b) State the value of for which the system of equations has no solution and explain the geometric interpretation of this. (2 marks)

|  |
| --- |
| Solution |
| No solution when . *[ and have parallel normals.]*  With this value, the system represents two non-coincident parallel planes cut by a third plane. |
| Specific behaviours |
| ✓ states value  ü geometric interpretation |

Question 7 (6 marks)

The equations of three planes are , and , where and are integer constants.

Elimination can be used to reduce the system of equations to

(a) Determine any necessary restrictions on the value of and/or the value of for the system of equations to have

(i) a unique solution. (1 mark)

(ii) no solutions. (1 mark)

(b) For a particular value of and value of , the three planes intersect in a straight line. Determine the vector equation of this line. (4 marks)

Question 7 (6 marks)

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Elimination can be used to reduce the system of equations to

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(i) a unique solution. (1 mark)

|  |
| --- |
| **Solution** |
| Require . |
| **Specific behaviours** |
|  correct restriction |

(ii) no solutions. (1 mark)

|  |
| --- |
| **Solution** |
| Require and .  Hence and . |
| **Specific behaviours** |
| ✓ correct restrictions |

(b) For a particular value of and value of , the three planes intersect in a straight line. Determine the vector equation of this line. (4 marks)

|  |
| --- |
| **Solution** |
| Require , and let .  Using reduced equation (2) :  Using reduced equation (1) :  Hence |
| **Specific behaviours** |
| ✓ indicates required value of and value of   expresses and in terms of parameter   expresses in terms of parameter   correct vector equation (many alternatives exist...) |

Question 7 (7 marks)

Three planes have equations

where is a constant.

(a) Explain why the planes cannot intersect at a unique point when . (2 marks)

The acute angle between the planes and is , where .

(b) Determine the value of . (5 marks)

Question 7 (7 marks)

Three planes have equations

where is a constant.

(a) Explain why the planes cannot intersect at a unique point when . (2 marks)

|  |
| --- |
| **Solution** |
| Hence no solution to system as equations inconsistent and so planes cannot intersect at a point. |
| **Specific behaviours** |
| ✓ subtracts equations   explanation |

The acute angle between the planes and is , where .

(b) Determine the value of . (5 marks)

|  |
| --- |
| **Solution** |
| Let normals to planes be and : |
| **Specific behaviours** |
|  magnitudes of normals  ✓ dot product of normals   equation using scalar product   squares both sides   correct value of |

Question 4 (8 marks)

Two planes have equations and .

(a) Determine the Cartesian equation of a third plane that is perpendicular to these planes and passes through the point . (4 marks)

(b) Determine the point of intersection of all three planes. (4 marks)

Question 4 (8 marks)

Two planes have equations and .

(a) Determine the Cartesian equation of a third plane that is perpendicular to these planes and passes through the point . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ identifies normal vectors to both planes   uses cross product   uses point   correct equation |

(b) Determine the point of intersection of all three planes. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ eliminates variable   eliminates same variable   solves for first variable   states solution |

Question 5 (5 marks)

A plane passes through the points , and . Determine the Cartesian equation of the plane.

Question 5 (5 marks)

A plane passes through the points , and . Determine the Cartesian equation of the plane.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ determines two vectors in plane  ✓ indicates use of cross product to find normal  ✓ correct normal  ✓ determines constant  ✓ states equation in Cartesian form |